$^{1}/_{n}$ Octave Analysis – FFT – Wavelet

For most acoustic examinations, a simple sound level analysis is insufficient, as not only the overall sound pressure level, but also the frequency-dependent distribution of the level has a significant influence on the perception of a sound event. This Application Note presents several analysis types for frequency-dependent sound level determination: the analysis with a filter bank of $^{1}/_{n}$ octave filters, the Fast Fourier Transformation and the Wavelet analysis. The functional principles of these analysis functions as well as their advantages and disadvantages are explained. In this regard, not only the frequency resolution, but – of course – also the time resolution of the respective analysis plays an important role. Therefore, the following describes especially those analysis functions that are displayed as a function of time.

$^{1}/_{n}$ Octave Analysis

In the $^{1}/_{n}$ octave analysis, the signal to be analyzed is split into partial signals by a digital filter bank before the sound level is determined. The filter bank consists of several filters connected in parallel, each with a bandwidth of $^{1}/_{n}$ octave. An octave filter is a filter whose upper cutoff frequency is twice the lower cutoff frequency, whereas 3rd octave filters further subdivide each octave band into three parts. This means that octave filters or $^{1}/_{n}$ octave filters don't have a constant absolute bandwidth,
but a constant relative bandwidth, i.e. the frequency bands are equidistant on a logarithmic frequency scale. Displayed on a linear frequency scale, the filter bandwidth increases logarithmically (see figure 1 below).

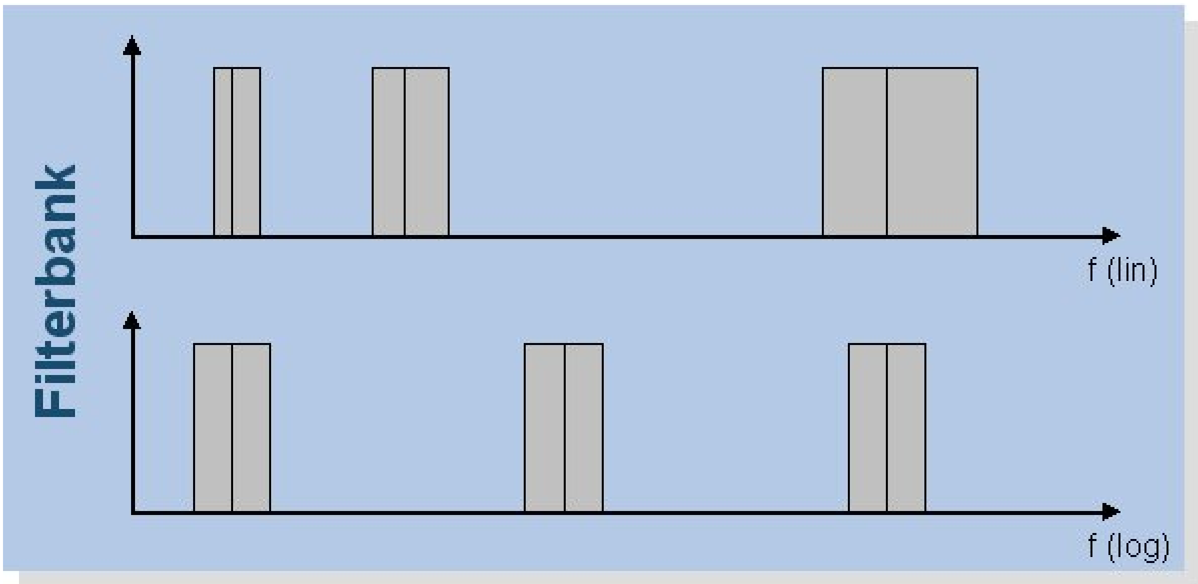

Figure 1: Schematic representation of the filter bandwidth on a linear and a logarithmic frequency scale

An octave filter with a center frequency of 63 Hz has a bandwidth of 44 Hz, whereas a 3rd octave filter with the same center frequency has a bandwidth of only 14.5 Hz. Towards higher frequencies, the bandwidth of these filters increases more and more. An octave filter with a center frequency of 16000 Hz has a bandwidth of 11360 Hz. A 3rd octave filter with this center frequency has a bandwidth of 3650 Hz.
The effects the filters broadening towards higher frequencies have on the analysis results are described below by means of white and pink noise. White noise has a frequency-independent spectral power density, i.e. the signal level is constant across the entire frequency range. When a white noise is analyzed with a filter bank of $^1/_n$ octave filters, the diagram shows a rising curve, because the filter bandwidth and thus the amount of energy per filter band increases towards higher frequencies (see figure 2, left). When the same filter bank is used to examine a pink noise signal whose amplitude drops by 3 dB per octave towards higher frequencies, the power is approximately the same for each filter band. The power drop of the pink noise towards higher frequencies is compensated by the increasing filter bandwidth. The $^1/_n$ octave analysis of pink noise therefore shows a frequency-independent curve (see figure 2, right).

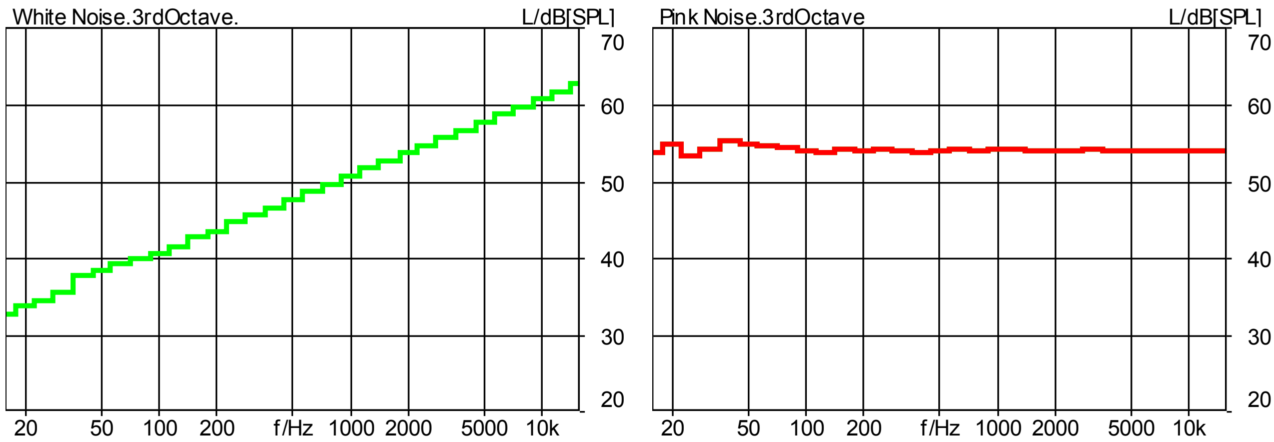

Figure 2: 3rd octave band analysis of white noise (left) and pink noise (right)

As the result of an $^1/_n$ octave analysis, either an averaged spectrum or a time- or RPM-dependent spectrogram can be calculated. For this purpose, ArtemiS provides the analysis functions “1/nth Octave”, “1/nth Octave (Peak Hold)”, “1/nth Octave vs. RPM” and “1/nth Octave vs. Time”.
Figure 3 shows the Properties dialog of the “1/nth Octave vs. Time” analysis. In this window, the user can select the various settings for the analysis. In the first selection box of the Properties dialog, the bandwidth of the filters is specified (“Band resolution”). Filters for octaves or for fractions thereof can be selected. To mimic human perception, 3rd octave filters are especially suited, because their bandwidth above 500 Hz approximately corresponds to the frequency groups used in psychoacoustics.
Besides the filter bandwidth, the frequency position of the filters can be adjusted as well. In the “Row” selection box, one of two filter rows can be selected, which are offset from each other by one half of the bandwidth. Row “B” is defined so that the 1 kHz mark corresponds to the center frequency of one filter. With the setting “A”, the 1 kHz mark corresponds to the cutoff frequency of one filter. Normally, row “B” is used.
In the selection box “Spectral Weighting”, a weighting of the analysis results with an A, B, C or D filter can be activated. With these filters, different frequency areas are weighted differently. The A

weighting is often used for the analysis of airborne sound signals. This weighting takes the frequency-dependent sensitivity of the human ear into account, because the A filter approximately corresponds to the inverted hearing threshold level of a person with normal hearing capabilities.

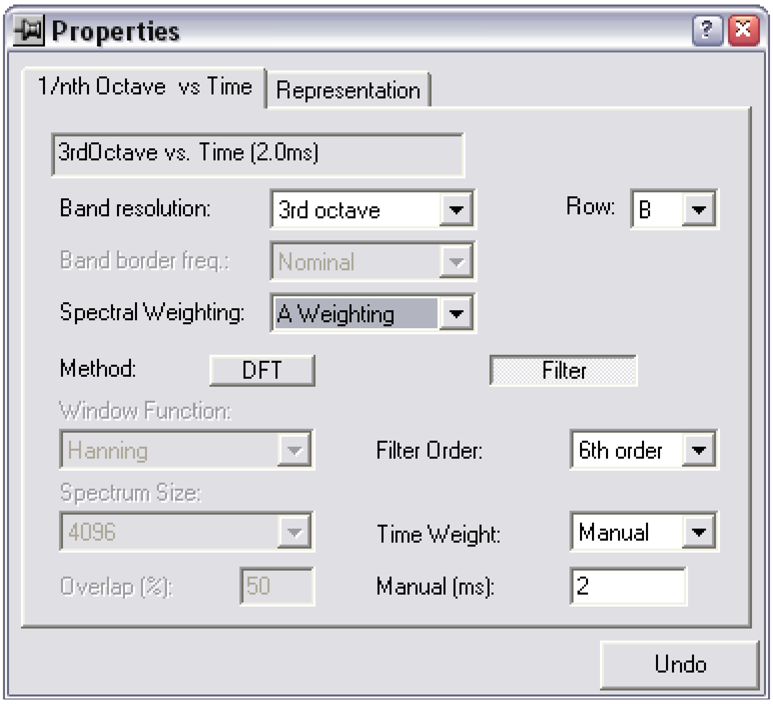

Figure 3: Properties dialog of the 1/nth Octave vs. Time analysis

In the Properties dialog shown in the figure, the Filter[1] method has been selected for the calculation of the analysis. In this method, the partial bands are extracted by filtering using an array of digital filters. By selecting the Filter method, the "Filter Order" field becomes available. Filters of fourth and sixth order are available. The sixth order filters comply with the ANSI standard S 1.11 and with IEC 1260. The 3rd octave filters of fourth order comply with the (outdated) DIN 45652. In the DFT method, the partial bands are calculated by adding the respective spectral lines of a DFT spectrum. Unlike the continuous processing of the signal in the Filter method, the DFT method processes the signal block wise, i.e. before analyzing the signal is sectioned into blocks of the length selected in the field "Spectrum size" ("windowing").Since the Filter method does not require a windowing of the signal, it is preferable to the DFT method.
If the Filter method is selected, the Properties dialog also offers the possibility to specify a "Time Weighting". This function smoothes the level curves by means of exponential integration. Available options are "Slow", "Fast", "Impulse" and "Manual". The "Slow" integration period is 1 second and the "Fast" period is 125 ms. In the "Impulse" setting, a peak detector with an integration period of 35 ms and a decay time of 1500 ms is activated. If "Manual" is selected, the integration period can be specified by the user in the field "Manual (ms)". The longer the selected integration period, the more the level curve is smoothed. To resemble the processing time of human hearing, an integration period of 2 ms should be used.

[1] This method can only be selected if the ArtemiS license in use includes ATP 04, otherwise only the DFT method is available.

Fast Fourier Transformation

The Fourier analysis is based on a mathematical theorem introduced by J. B. Fourier. The statement of this theorem can be summarized as follows: Any periodic signal form can be interpreted as a superposition of discrete, periodic sine and cosine oscillations with different frequencies and amplitudes. Practical implementations of this theorem can be found in the Discrete Fourier Transformation (DFT) and the Fast Fourier Transformation (FFT). The results of the DFT and FFT provide discrete frequency spectra of a sampled time domain signal. The FFT is a variant of the DFT requiring less computing resources.
Before the transformation, the signal has to be sectioned (windowed) on the time axis, therefore the original signal is subdivided into several blocks with N samples each. This is the so-called windowing. In an averaged analysis, "FFT(Average)", the results of the individual blocks are averaged. In the time-dependent analysis variants, "FFT vs. Time" and "FFT vs. RPM", the results of the individual blocks are displayed successively in a spectrogram. An averaged analysis is suitable for stationary signals, whereas the time-dependent analysis is the method of choice for non-stationary signals. Unlike the 3rd octave level analysis, the FFT is an analysis with a constant bandwidth. Figure 4 schematically shows the distribution of the frequency nodes of the FFT analysis on a linear and on a logarithmic frequency scale. The frequency nodes on the linear frequency scale are equidistantly distributed, whereas on the logarithmic frequency scale they are located closer to each other with higher frequencies.

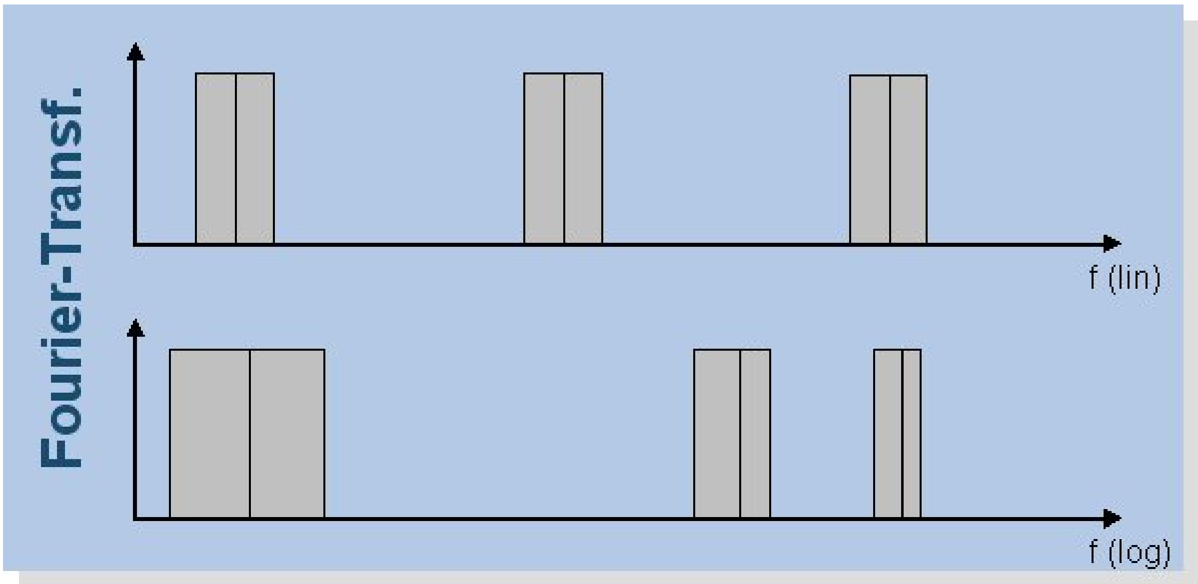

Figure 4: Schematic representation of the frequency nodes of the FFT analysis on a linear and a logarithmic frequency scale

Due to of the constant distribution of the nodes the FFT analysis of white noise shows a frequency-independent curve, while the analysis of pink noise shows a descending curve (see figure 5).

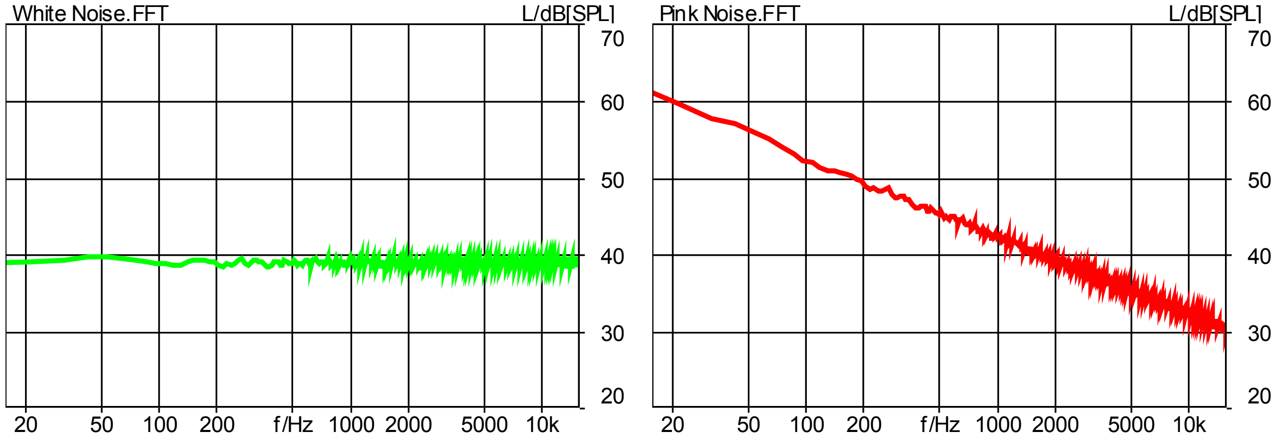

Figure 5: FFT analysis of white noise (left) and pink noise (right)

Figure 6 shows the Properties dialog of the “FFT vs. time” analysis which is described in detail below. In the selection box “Spectrum Size”, the block length for the analysis is selected. A large block length results in a high frequency resolution of the FFT analysis, whereas a small block length leads to a higher time resolution. Due to the temporal windowing, the FFT analysis is subject to a time/frequency blurring: High frequency resolution always results in low time resolution and vice versa. However, the possibility to specify the window size allows the user to choose which of the two aspects is more important for his application. At a sampling rate of 44100 Hz, a block length of 4096 samples results in a time resolution of 0.093 seconds. The frequency resolution in this case is 10.77 Hz. By increasing or decreasing the block length, the frequency resolution or the time resolution is improved respectively.

In the next selection box, the desired “Window Function” can be selected. The window function allows a time weighting of the individual block sections in order to reduce the so-called leakage effect. As already described, the signal must be cut into a number of blocks for the FFT analysis. In the analysis of these blocks, a periodic continuation of the signal is assumed. This can lead to points of discontinuity at the borders of the signal section if no integer multiple of the period is contained in the block. These discontinuities result in frequencies in the spectrum that don’t exist in the original signal. This "leakage" of signal energy into neighbor frequencies of the original frequency is what gives the leakage effect its name. By means of suitable windowing with window functions that go to zero at their borders, this effect can be reduced. Since the selection of the window function affects the analysis result, a window function suitable for the respective application must be selected. For many applications, the Hanning window is a good choice, as it greatly reduces the leakage effect. Other window functions are optimized for specific applications. For example, the Kaiser-Bessel window has a very good frequency resolution and should be used if separate tonal components with very different levels are to be separated from each other.

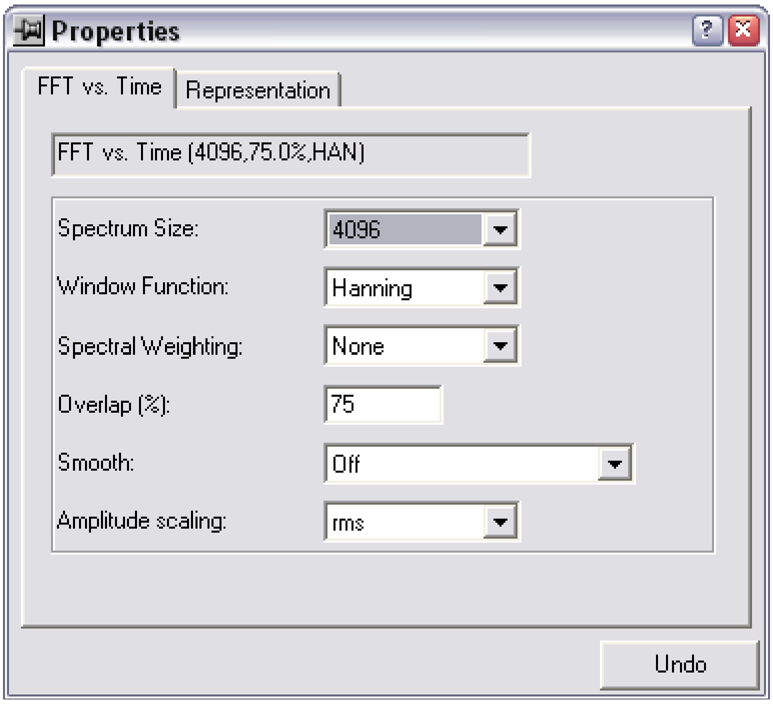

Figure 6: Properties dialog of the “FFT vs. Time” analysis

To compensate for the time weighting of the window functions, an overlapping of the windows can be specified. In the field “Overlap [%]”, the desired overlapping can be entered in percent. For the Hanning window, overlappings of 50 % or 66.67 % are often used. An overlapping of 50 % results in a windowing compensation with accurate amplitudes, whereas an overlapping of 66.67 % keeps the signal power constant.

In the selection box “Spectral Weighting”, the user can activate or deactivate a frequency-dependent weighting. Just like for the $^1/_n$ octave analysis, an A, B, C or D weighting can be selected.

With the “Smooth” function, the calculated FFT spectrum curve can be smoothed. Smoothing can be used if the spectral distribution of a signal is to be calculated, for example for the creation of a non-recursive filter (FIR filter) based on these results.

In the selection box „Amplitude scaling“ two possibilities are available: „rms“ and „peak“. In the first case each FFT line displays the effective value of the oscillation, in the second case the peak value is calculated, which is $\sqrt{2}$ times higher than the rms value (for sinus oscillations).

In the last row of the Properties dialog it can be specified if and how the phase is to be determined in the calculation. It to be considered that for this calculation the representation settings on the second tab of the Properties dialog have to be changed (e.g. switch from “Default” to “Amplitude/Degree” and select complex representation in two diagrams.)

Wavelet

In the Wavelet analysis, the sound signal is examined using small wave packets, so-called *Wavelets*. For this purpose, ArtemiS uses the impulse responses of different bandpass filters as Wavelet analysis functions.

Unlike the FFT with its constant analysis bandwidth, the Wavelet analysis (just like the $^{1}/_{n}$ octave analysis) has a frequency resolution with a constant relative bandwidth. Figure 7 illustrates the difference between the frequency and time resolution of the FFT and Wavelet analysis.
The X axis in the drawing represents time, the Y axis represents frequency. A narrow, high box stands for a high time resolution and a low frequency resolution, whereas a flat, wide Box symbolizes a good frequency resolution and a low time resolution. Due to the time-frequency blurring, the area of each of these "time-frequency" boxes in the figure is the same.

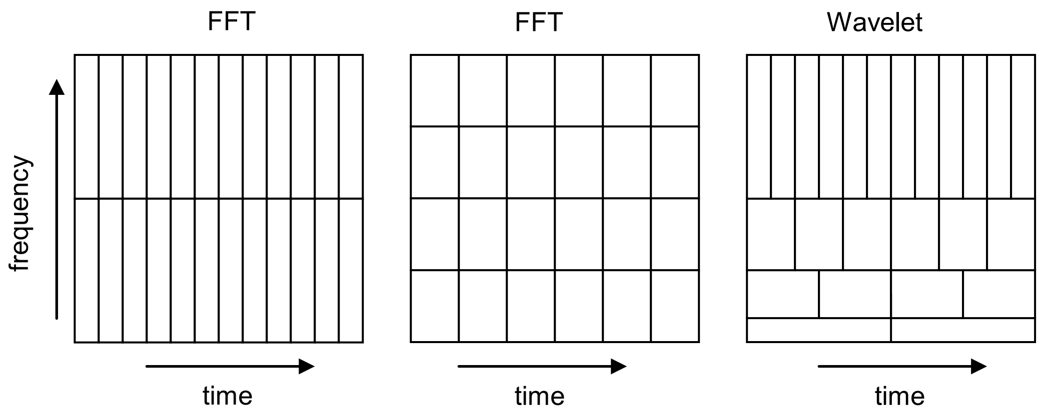

Figure 7: Frequency and time resolution of FFT and Wavelet analysis

The left and center diagrams show the resolution of a FFT analysis with different window lengths. The left diagram shows the resolution of an FFT analysis with a small window length (i.e. a high time resolution), resulting in a low frequency resolution. The FFT in the center diagram has a larger window length, which reduces the time resolution, but improves the frequency resolution. Both the time resolution and the frequency resolution of the FFT are constant across the entire frequency range. This is not the case with the Wavelet analysis, as shown in the right diagram of figure 7. At low frequencies, the Wavelet analysis delivers a high frequency resolution combined with a low time resolution. Towards higher frequencies, the frequency resolution gets worse, but the time resolution improves significantly. The area of the boxes always remains constant.
The resolution of the Wavelet analysis is a good approximation of the analysis that takes place in the human ear.

Figure 8 shows the Properties dialog of the Wavelet analysis in ArtemiS. Just like for the other analysis types, the first selection box allows a spectral weighting to be specified, which weights the calculated spectrum using an A, B, C or D filter.
The next section contains parameters for the bandpass filter whose impulse response is to be used as the Wavelet analysis function. These parameters include the filter type, the filter order and the Q value (“Filter Quality“). Thus the rate of rise of the filter edges and the filter bandwidth are specified. A steep, narrow filter means a high frequency-resolution, but also a low time resolution (and vice versa).
By specifying the “Frequency Range” to be analyzed, the examination can be limited to the interesting area. In the last selection box of the Properties dialog, the required “Resolution” can be specified. If the setting “High” is selected, 128 bandpass filters are used for the analysis of the specified frequency range, 64 filters are used for the “Medium” resolution and 32 filters for the

“Low” resolution. A higher number of bandpass filters not only causes a better resolution, but also a longer computing time.

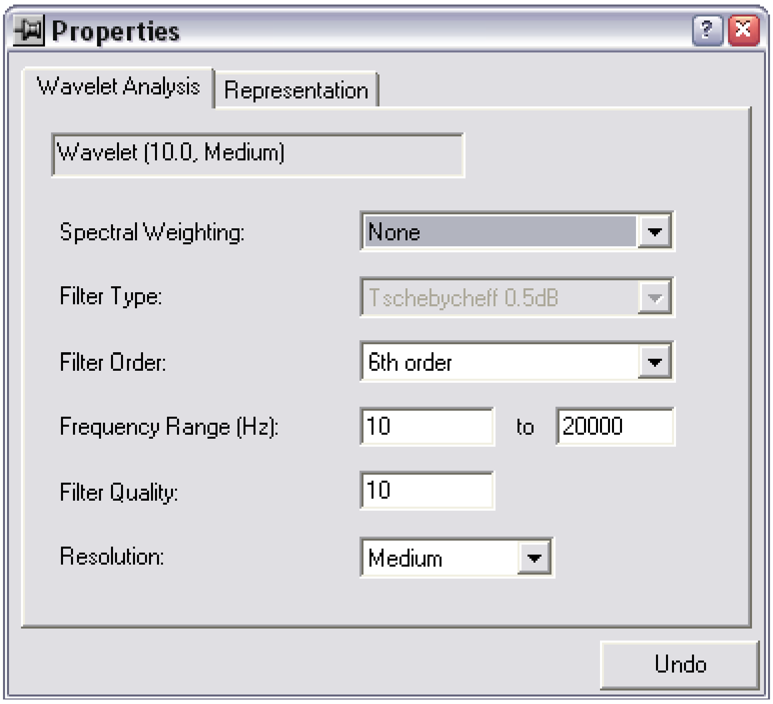

Figure 8: Properties dialog of the Wavelet analysis

Just like the $^1/_n$ octave analysis, the Wavelet analysis in ArtemiS performs a time weighting by means of an exponential integration. However, the Wavelet analysis does not use a fixed, but a frequency-dependent time constant T defined as follows:

$$T = \frac{1}{\text{center frequency of the bandpass filter}}$$

Comparison of the Analysis Methods

In the following, two sounds from the automotive area are to be analyzed: One is the sound of a car door being closed, the other is an engine sound with a clearly noticeable high-frequency whistling of the turbocharger. The car door generates a very short, broadband noise, whereas the engine noise contains a very distinct tonal component. For a meaningful analysis of these sounds, it is not only necessary to choose the right analysis type, but also selecting the correct analysis parameters. It must be considered that the correct analysis parameters not only depend on the type of sound, but also on the specific aspect of the sound that is to be examined in detail. For the analysis of the door closing sound, this means that even though the sound is very short, a large block length may be required for the FFT analysis if the individual frequencies of the sound are to be examined.

The figures below illustrate the difference between the analysis types and the effect of changing the parameter settings. Figures 9 and 10 shows analysis results of the door closing sound. The two diagrams on the left side show a $^1/_n$ octave analysis, the center diagrams shown an FFT analysis, and the right diagrams show a Wavelet analysis. The difference between figure 9 and

10 is one respective parameter that has been changed for each of the calculations. The following analysis parameters were used:

$1/n^{th}$ Octave vs. time:

Band Resolution:	3^{rd} Octave (fig. 9) / 12^{th} Octave (fig. 10)
Spectral Weighting:	None
Method:	Filter
Filter Order:	6th Order
Time Weighting:	Manual
Manual (ms):	2

FFT vs. time :

Spectrum Size:	512 (fig. 9) / 4096 (fig. 10)
Window Function:	Hanning
Spectral Weighting:	None
Overlap (%):	75
Smooth:	Off

Wavelet:

Spectral Weighting:	None
Filter Type:	Tschebycheff 0.5 dB
Filter Order:	6th Order
Frequency Range:	20 to 16000 Hz
Filter Quality:	10
Resolution:	Medium (fig. 9) / High (fig. 10)

The comparison of the diagrams in figure 9 shows that the frequency resolution of the different analysis types is different. In the FFT analysis, the frequency resolution is constant across the entire frequency range. However, since the frequency axis is shown logarithmically, the frequency resolution of the FFT analysis appears to be worse at low frequencies than at high frequencies. The 3rd octave level analysis (fig. 9a), just like the Wavelet analysis (fig. 9c) has a frequency resolution that is constant on the logarithmic frequency axis. Furthermore, figure 9 shows that the results of those analysis types based on digital filtering (3rd octave level and Wavelet analysis) have a slight delay at low frequencies. This is caused by the tuning process of the digital filters, which takes longer for low-frequency filters. The FFT analysis does not involve such a tuning, therefore in its diagram the display of the low frequencies is not shifted.
The difference between the diagrams in figure 9 and 10 is the higher frequency resolution in figure 10. The improvement of the frequency resolution, however, also has negative effects. In the $^1/_n$ octave analysis (figure 9a and 10a), the increasing of the resolution from 3rd octaves to 12th octaves leads to a stronger temporal smearing of the analysis result at low frequencies. In the FFT analysis (figure 9b) and 10b), the comparison shows that the better frequency resolution (which is best visible at low frequencies) leads to a very limited time resolution. In the Wavelet analysis, the effect of the changed analysis parameters is only marginal.

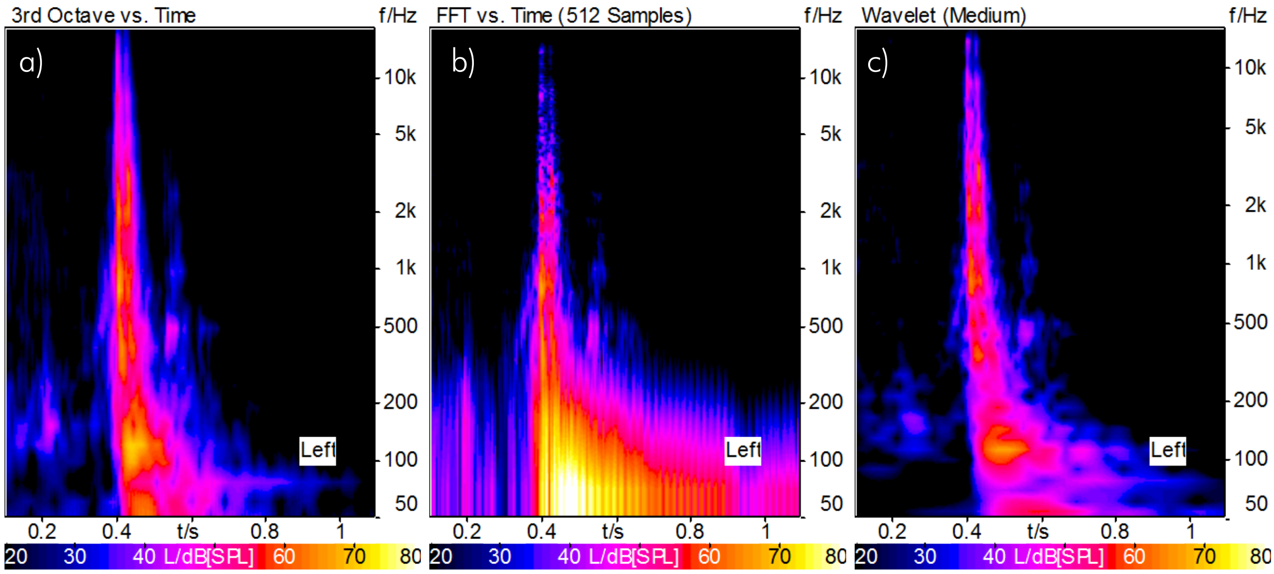

Figure 9: Analysis result of a door closing sound; left diagram: 3rd octave level analysis, center diagram: FFT analysis (block length: 512 samples), right diagram: Wavelet analysis (resolution: Medium).

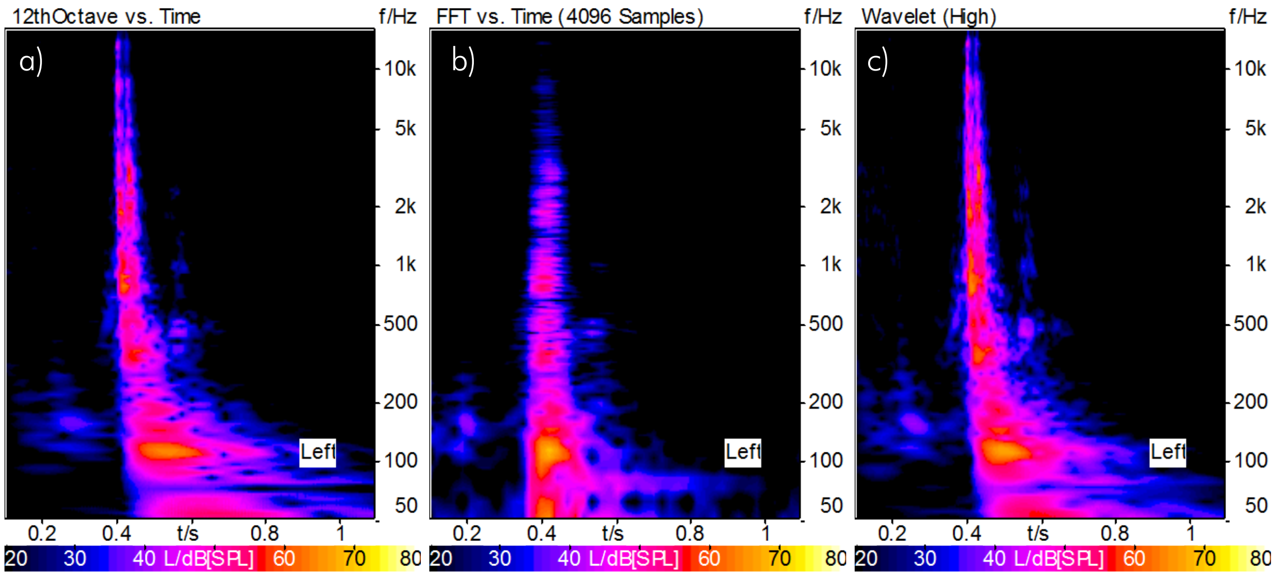

Figure 10: Analysis result of a door closing sound; left diagram: 12th octave level analysis, center diagram: FFT analysis (block length: 4096 samples), right diagram: Wavelet analysis (resolution: High).

Altogether, the Wavelet analysis represents the frequency-time progression of the door closing sound very well. However, the other two analysis types reveal important details as well if the strengths and weaknesses of the respective analysis type are taken into account. Which analysis is eventually used depends, of course, also on the user's personal preferences and practice.
One advantage of the FFT analysis not mentioned yet is the much shorter calculation time compared to the Wavelet analysis. If quick analysis results of a large amount of data are needed, this would be a job for the FFT method.

Figure 11 shows the analysis results of an engine sound containing a tonal whistling component. For the analysis, the same parameters were used as for the door closing sound in figure 10. The comparison between figure 11a, 11b and 11c shows that for the analysis of this high-frequency tonal component, the FFT analysis with a block length of 4096 samples is especially well suited.

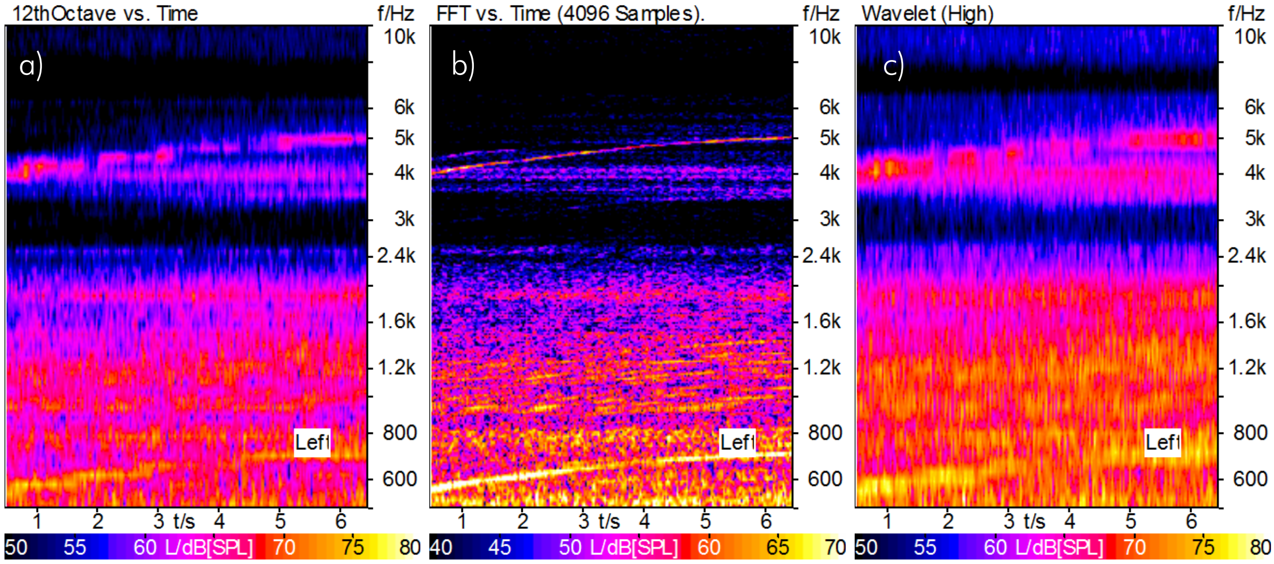

Figure 11: Analysis result of an engine sound; left diagram: 12th octave analysis, center diagram: FFT analysis (block length: 4096 samples), right diagram: Wavelet analysis (resolution: High).

The whistling of the turbocharger between 4 and 5 kHz can be seen very clearly in this analysis. In the two other analysis methods, the frequency resolution at high frequencies is not sufficient to resolve the tonal component.

Note

In order to use the features presented in this Application Note you will need the ArtemiS Basic Version (Code 4600) and the ArtemiS Tool Packs ATP 04 (Code 4604, enabling digital filters for 1/n octave analyses) and ATP 07 (Code 4607, providing the wavelet analysis).

Do you have questions for the author? Please contact us at imke.hauswirth@head-acoustics.de.
We look forward to your feedback!